Todd is a testing supervisor in a factory that produces and packages small metal parts. A packaging machine is set to drop 500 nails into each box. However, the nails vary slightly in weight. A 5-pound box of nails is approved for shipping if its weight is between 78.5 ounces and 81.5 ounces.

### Compound Inequalities

Inequalities are often combined into one statement. Consider the following two inequalities.

\[
2 < 3 \quad \text{and} \quad 4 < 5
\]
\[
7 > 4 \quad \text{or} \quad 6 > 1
\]

The first inequality states that “2 is less than 3 and 4 is less than 5.” The second states that “7 is greater than 4 or 6 is greater than 1.” Such inequalities are called **compound inequalities**.

### Example 1  Writing Compound Inequalities

Refer to the information in the opening paragraph of this lesson. Write a compound inequality to show the weight approved for shipping a 5-pound box of nails.

**Solution**

Let \( x \) represent the weight of the nails and box. Combine the two inequalities \( 78.5 < x \) and \( x < 81.5 \).

The compound inequality is \( 78.5 < x < 81.5 \). The inequality is read “78.5 is less than \( x \) and \( x \) is less than 81.5.”

### Conjunctions

A compound inequality that uses the word *and* to separate two simple inequalities is an example of a **conjunction**. The solution set of a conjunction is the set of solutions that make both simple inequalities true. This solution is the intersection of the solutions of the simple inequalities.
Example 2  Solving a Conjunction

Solve and graph \(-3 < (2x - 3) \leq 5\).

Solution

Write the simple linear inequalities separately. Solve each inequality.

\[-3 < (2x - 3) \quad \text{and} \quad (2x - 3) \leq 5\]

\[-3 + 3 < (2x - 3) + 3 \quad \text{and} \quad (2x - 3) + 3 \leq 5 + 3\]

\[0 < 2x \quad \text{and} \quad 2x \leq 8\]

\[0 < x \quad \text{and} \quad x \leq 4\]

The solution is the set of all numbers greater than zero and less than or equal to four, which is \(0 < x \leq 4\).

To graph the conjunction, graph the common solution to the two simple inequalities on the same number line.

The solution set for the conjunction is the intersection of the solution sets of the two simple inequalities. This is shown as the shaded segment that is common to both graphs.

Steps to Check Solutions for a Conjunction

1. Choose a value in the intersection and substitute it into each simple inequality. The value should make both of the original inequalities true.
2. Choose a value to the left of the interval. This value should make one of the original inequalities true.
3. Choose a value to the right of the interval. This value should make the other simple inequality true.

Ongoing Assessment

Solve and graph each conjunction. Check your solution set.

a. \(m\) is greater than \(-5\) and less than \(4\)

b. \(-2 \leq 3y + 1 \leq 5\)

Critical Thinking  How can you check the solution set of a conjunction by using the endpoint values of each simple inequality?
The solution set of a conjunction is the intersection of two sets of numbers. The intersection of two sets consists of the numbers that are common to both sets; in other words, the numbers in one set that are also in the other set. A Venn diagram gives a pictorial representation of the intersection of two or more sets of data. The shaded area in the Venn diagram below represents the intersection of sets A and B. The intersection of the two sets is designated $A \cap B$.

$$A \cap B = \{3, 8\}.$$  

Because the elements 3 and 8 are in the shaded (overlap) region,

$$A \cap B = \{3, 8\}.$$  

Consider again the combined inequality in Example 2.

$$-3 < (2x - 3) \leq 5$$

For the left inequality, $-3 < 2x - 3$, the solution is the set of all values for which $x > 0$. Call these values Set A.

For the other inequality, $2x - 3 \leq 5$, the solution is the set of all values for which $x \leq 4$. Call these values Set B.

For the combined inequality, the solution is $0 < x \leq 4$.

The intersection of Sets A and B (or $A \cap B$) is pictured below.
Activity 1 The Empty Set

1. Solve and graph \(-3 > 2x - 3\).
2. Solve and graph \(2x - 3 \geq 5\).
3. What is the solution to \(-3 > 2x - 3\) and \(2x - 3 \geq 5\)?
4. Use a Venn diagram to picture the solution to the conjunction.
5. Explain why the solution is called the empty set.

Disjunctions

Wire stock used to make nails is tested on a machine that compresses the wire and then stretches it. An 8-inch length of wire fails the test if it is compressed to a length shorter than 7.95 inches or stretched to a length longer than 8.05 inches. A compound inequality models the wire length that fails the test:

\[ x > 8.05 \text{ or } x < 7.95, \text{ where } x \text{ is the length of the nail wire.} \]

This compound inequality consists of two simple inequalities connected by the word or. It is an example of a disjunction. The solution set of the disjunction is the set of all solutions that make either of the simple inequalities true.

Example 3 Graphing a Disjunction

Solve and graph \(-3 > 2x - 3\) or \(2x - 3 \geq 5\).

Solution

Solve each simple linear inequality.

\[-3 > (2x - 3) \quad \text{or} \quad (2x - 3) \geq 5\]
\[-3 + 3 > (2x - 3) + 3 \quad \text{or} \quad (2x - 3) + 3 \geq 5 + 3\]
\[0 > 2x \quad \text{or} \quad 2x \geq 8\]
\[0 > x \quad \text{or} \quad x \geq 4\]

Graph the two inequalities on a number line.
All the points for $0 > x$ and $x \geq 4$ constitute the solution for the combined inequality connected by the word *or*.

To check the solution to the disjunction, check $x$-values less than zero and $x$-values greater than 4 to show that the values make the combined inequality true. Then check $x$-values greater than or equal to zero and less than 4 to show that these values do not satisfy either inequality.

**Ongoing Assessment**

Solve and graph $8y + 1 < 25$ or $-2y < -10$.

**Union**

The solution set of a disjunction is the *union* of two sets of numbers. The union of two sets of numbers consists of all the numbers that are in either one of the sets. The shaded area in the Venn diagram below represents the union of sets $A$ and $B$. The union is designated $A \cup B$.

\[
\text{Set } A = \{1, 2, 3, 6, 7, 8\} \quad \text{Set } B = \{3, 4, 5, 8, 9\}
\]

Thus, the union of the two sets $A$ and $B$, designated as $A \cup B$, is the set $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

**Activity 2**

**Graphing a Subset**

1. Solve and graph $5y - 2 < 8$.
2. Solve and graph $3y + 4 < 16$.
3. What is the solution to $5y - 2 < 8$ or $3y + 4 < 16$?
4. Use a Venn diagram to picture the solution to the disjunction.
5. Explain why the solution for $5y - 2 < 8$ is called a *subset* of the solution for $3y + 4 < 16$.

**Critical Thinking** In the language of algebra, *and* statements correspond to the intersection of two sets, and *or* statements correspond to the union of two sets. How is this similar to the way the words are used in probability?
**Lesson Assessment**

**Think and Discuss**

1. What steps are the same when you solve a conjunction and a disjunction? What steps are different?

2. How is the word *and* used to find the solution of a conjunction? How is the word *or* used to find the solution to a disjunction?

3. How do you know if a compound inequality is a conjunction or a disjunction?

**Practice and Problem Solving**

Solve each compound inequality. Graph the solution.

4. $2x \leq 6$ or $x > 5$

5. $3x < 2.4$ and $x \geq -1.5$

6. $2x - 3 < 13$ and $x > 4$

7. $5x + 4 < -11$ or $3 - 4x < -1$

8. $3x + 1 > 10$ or $2x - 5 < 3$

9. $x + 3 < -1$ and $4x - 1 < 5$

10. $-5 < 7 - 2x < 5$

11. $3 < 4x + 7 < 15$

Write and solve a compound inequality for each situation.

12. Tad will spend at least $20 but no more than $50 on T-shirts for summer camp. The cost of a T-shirt is $8. How many T-shirts can Tad buy?

13. A thermostat will send an electrical signal if it senses a temperature below 65 degrees or above 80 degrees. If the initial temperature increases by 8 degrees, the thermostat will send a signal. What are the possible initial temperatures?

14. In Exercise 13, if the initial temperature decreases by 8 degrees, the thermostat will send a signal. What are the possible initial temperatures?
15. A store owner adds $5 to twice the wholesale price in order to set the retail price of his sweaters. If the retail prices are between $145 and $155, what wholesale prices did the store owner pay for the sweaters?

Mixed Review

16. The heights in inches and weights in pounds of 12 students are recorded.

<table>
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<th>68</th>
<th>71</th>
<th>70</th>
<th>69</th>
<th>72</th>
<th>70</th>
<th>69</th>
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<th>65</th>
<th>65</th>
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</thead>
<tbody>
<tr>
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<td>160</td>
<td>140</td>
<td>135</td>
<td>139</td>
<td>150</td>
</tr>
</tbody>
</table>

a. Find the mode of the heights.
b. Find the median of the weights.
c. Make a scatter plot from the data. Let height be the independent variable.
d. Describe the correlation between the two sets of data.

Solve each system of equations.
17. \(3a + 2b = 0\) \(a + 3b = -7\)
18. \(4c - 5d = -8\) \(6c + 2d = 7\)

Write and solve a system of equations that models each situation.
19. Mario has saved $20 less than Nancy. Together they have saved $100. How much has each person saved?
20. A chemist needs 400 grams of a 5% salt solution. There is one available solution containing 4% salt and another available solution containing 8% salt. How many grams of each solution should the chemist mix to get the desired 5% solution?
21. Mal bought 15 gallons of 87 octane gas and 12 gallons of 92 octane gas for $77.40. Yuri bought 10 gallons of 87 octane gas and 7 gallons of 92 octane gas for $48.65. How much does a gallon of each type of gas cost?